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Survival probability and field theory in systems with absorbing states

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An important quantity in the analysis of systems with absorbing states is the survival probability $P_s(t)$, the probability that an initial localized seed of particles has not completely disappeared after time t. At the transition into the absorbing phase, this probability scales for large t like $t^{-\delta}$. It is not at all obvious how to compute δ in continuous field theories, where $P_s(t)$ is strictly unity for all finite t. We propose here an interpretation for δ in field theory and devise a practical method to determine it analytically. The method is applied to field theories representing absorbing-state systems in several distinct universality classes. Scaling relations are systematically derived and the known exact δ value is obtained for the voter model universality class. [S1063-651X(97)00211-0]

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I. INTRODUCTION

Certain stochastic nonequilibrium systems possess absorbing configurations, that is, states that have no fluctuations, and in which the system can become trapped. Among the many examples are autocatalytic chemical reactions [1], the contact process [2,3], directed percolation [3,4], the voter model [5], models for the spread of epidemics or forest fires [6,7], systems with multiplicative noise [8], and models of transport in random media [9]. The number of absorbing configurations is typically unity, but can be larger than that [10], and in some cases can diverge in the thermodynamic limit [11].

The phase diagram of this kind of system consists, in general, of two different phases: an *absorbing phase*, in which the steady state consists entirely of absorbing configurations and in which the order-parameter field vanishes identically, and an *active phase*, in which the steady-state dynamics is nontrivial and the order-parameter field has a nonvanishing expectation value. Separating these two phases is a *critical point* (or surface of critical points), where the system exhibits a nonequilibrium phase transition from the active to the absorbing phase. As usual, physical quantities behave like power laws at the critical point. An important task is the categorization of different discrete lattice models with absorbing states into universality classes characterized by specific sets of critical exponents. Typically this is done by associating with each universality class a field theory with unique symmetry properties.

Standard quantities of interest such as the order parameter $M(\Delta)$, the correlation length $\xi(\Delta)$, and the correlation time $\tau(\Delta)$ are computed in steady state as functions of the distance Δ to the critical point. Typically, such computations begin with homogeneously random initial conditions. In systems with absorbing states, however, the dynamical evolution from an initial condition consisting of an absorbing configuration slightly modified by a localized ''seed'' of the order parameter gives additional information about the critical point. For a concrete example of such a seed initial condition, consider models for the spread of epidemics. Here the seed consists of an isolated infected individual placed at some arbitrary site of a lattice otherwise populated by healthy individuals. The propagation of the infection is then studied as a function of time for different parameter values. The critical point or surface separates the active phase, in which the initial infected seed propagates indefinitely, from the absorbing phase, in which the infection dies out with probability one as $t\to\infty$.

Interesting quantities to consider in this case are $N(t)$, $R(t)$, and $P_s(t)$, respectively defined (in the language of epidemics) as the average total number of infected sites, the average linear extent of the infected region, and the survival probability, i.e., the probability that at time t the system has not reached an absorbing configuration free from infected sites [12]. At the critical point and for asymptotically long times, these quantities scale like $N(t)\sim t^{\eta}$, $R^2(t)\sim t^{z}$, and $P_s(t)\sim t^{-\delta}$, which define the exponents η, z, and δ, respectively.

While there has been steady progress in associating groups of discrete lattice models with particular field theories and hence a known universality class, an outstanding problem with defining the survival probability remains. The point is that for microscopic lattice models, where the dynamical variable is typically discrete, there is always a nonzero probability of reaching the absorbing state in finite time from a seed initial condition. $P_s(t)$ is then clearly a nonincreasing function of t, and the associated exponent δ can be readily defined at criticality. In field theories with continuous variables, on the other hand, the absorbing state is a set of measure zero in phase space and so can never actually be reached in finite time. Thus $P_s(t)$ is strictly equal to unity for all t, so the concept of the survival probability has no utility. It is not at all clear that δ can even be sensibly defined, let alone computed. Indeed, this exponent is not calculated directly from field theory in any of the existing analytical studies. Rather, its value is inferred from scaling laws relating it to other, calculable exponents [4].

II. δ EXPONENT IN FIELD THEORIES WITH ABSORBING STATES

We now propose a solution to this problem by showing how δ can be defined and calculated in field theory. The main idea is that, although $P_s(t)=1$, states arbitrarily close to the absorbing configuration can be reached in finite time. In order to gain intuition about the approach to the absorbing state in models with continuous variables, let us consider the stochastic partial differential equation (Langevin) formulation of the field theory [known as Reggeon field theory (RFT) [4]], that characterizes the directed-percolation universality class. Aside from directed percolation, this important class includes, among many other problems, the contact process and simple models of heterogeneous catalysis and epidemiology [3].

The Langevin equation takes the form

$$\partial\phi(\mathbf{x},t)/\partial t=\nu\nabla^2\phi+a\phi-b\phi^2+\sqrt{\phi}\,\eta(\mathbf{x},t), \tag{1}$$

where ν, a, and b are constants, $\phi(\mathbf{x},t)$ is a continuous, positive-semidefinite variable defined at position $\mathbf{x}$ and time t, and η is a Gaussian noise variable whose only nonvanishing correlations are $\langle\eta(\mathbf{x},t)\eta(\mathbf{x}',t')\rangle=D\delta(\mathbf{x}-\mathbf{x}')\delta(t-t')$ for some noise strength D. This stochastic process can, through standard techniques [13], be transformed into a Lagrangian formulation, the resulting field theory being RFT. It is clear from the Langevin equation that $\phi(\mathbf{x})=0$ is an absorbing state that persists indefinitely in time. Roughly speaking (i.e., at the mean-field level), this state is stable and unstable for $a<0$ and $a>0$, respectively.

For simplicity, let us now consider the zero-dimensional (0D) Langevin equation appropriate for a single variable $\partial\phi/\partial t=a\phi-b\phi^2+\sqrt{\phi}\eta$. It is a simple exercise to derive the (Fokker-Planck) equation for the evolution of the probability distribution function associated with this equation [14], and from it the stationary probability distribution function $P(\phi)$: $P(\phi)\propto(1/\phi)\exp[(2a\phi-b\phi^2)/D]$. $P(\phi)$ is nonnormalizable due to the nonintegrable singularity at the origin. This implies that the only stationary distribution is a δ function at $\phi=0$. (Since this is true for *any a* value, there is no active phase in the 0D problem.) An arbitrary initial probability distribution therefore evolves in time toward a distribution weighted at values of ϕ lying progressively closer to zero. Note, however, that ϕ cannot actually achieve the value 0 in finite time, so it is difficult to infer directly from the (Langevin) equation of motion that ϕ always reaches the absorbing state at $\phi=0$ asymptotically.

It is natural to suppose that a similar phenomenon occurs in higher dimensions, where active-to-absorbing phase transitions can occur. That is, any continuous equation in the same universality class as a microscopic model with $\delta>0$ should exhibit at criticality (and of course in the absorbing phase) a progressive accumulation or piling up of the probability distribution around the absorbing configuration [15]. Based on this notion, we propose to relate the exponent δ appearing in the microscopic models to the exponent governing the accumulation of the probability density in a neighborhood of the absorbing state of the corresponding Langevin equation or field theory. More specifically, we define $P(\alpha,t)$ for the field theory as the probability that the space integral of the density field is larger than an arbitrary constant α. Assuming that at the critical point there is a piling up of probability at the origin, $P(\alpha,t)$ for any fixed α will decrease with increasing t asymptotically. We then define δ_α at criticality through $P(\alpha,t)\sim t^{-\delta_\alpha}$, or, taking logarithmic derivatives, as

$$\delta_\alpha\equiv-\lim_{t\to\infty}\frac{\partial\ln[P(\alpha,t)]}{\partial\ln t}. \tag{2}$$

If c is a real number larger than one, then trivially $P(\alpha,t)>P(c\alpha,t)$, implying that $\delta_{c\alpha}\geqslant\delta_\alpha$. We now conjecture that

$$\delta=\delta_{c\alpha}=\delta_\alpha \tag{3}$$

for all sufficiently small α and $c\alpha$, and that δ defined in this way is the same as the survival probability exponent in the corresponding discrete models.

Though at present there is no direct numerical support for this conjecture, there is some indirect support. Following a strategy introduced by Dickman [16], we have numerically studied discretized versions of RFT, modified so as to produce an absorbing subspace A_ϵ. Here A_ϵ is defined as the subspace of states in the phase space of the discretized field theory in which the density field ϕ_i at every point i of the discrete spatial lattice is less than ϵ. If the system enters this subspace as it evolves in time under numerical simulation, then it is considered to have reached the absorbing state and the simulation is terminated. Both our simulations of this type and Dickman's results show that the numerical value of δ_ϵ is independent of ϵ in such models, and that this value coincides with the value obtained from microscopic models, such as the contact process [2], believed to be in the same (directed-percolation) universality class [17] as RFT. In the following sections, we use our conjecture to compute δ for various field theories with absorbing states.

III. GENERAL PROCEDURE

Computing δ requires the evaluation of $P(\alpha,t)$ for the problem with seed initial conditions. For any field theory, standard methods allow one to express $P(\alpha,t)$ in a path integral representation as [13]

$$P(\alpha,t)=\int D\phi\,D\psi\,\Theta\left(\int d^dx\;\phi-\alpha\right)\frac{\exp(-S)}{Z}\,\psi(\mathbf{x}_0,t_0). \tag{4}$$

Here ϕ is the density field, ψ is the response field, Θ is the Heaviside step function, $S=-\int dt\,\mathcal{L}$ is the action, i.e., the time integral of the dynamical Lagrangian defining the field theory, and the normalization factor Z is the partition function of the associated $(d+1)$-dimensional problem. The interpretation of the different terms in Eq. (4) is as follows: $\psi(\mathbf{x_0},t_0)$ creates a perturbation at position $\mathbf{x_0}$ and time t_0, representing the initial seed [13]. Subsequently, the average of $\Theta(\int d^dx\;\phi(\mathbf{x},t)-\alpha)$ is computed by evaluating all contributions coming from all possible paths starting at time t_0 and ending at time $t>t_0$. Each path is weighted by the exponential of its associated action, properly normalized. Since $\Theta(\int d^dx\;\phi(\mathbf{x},t)-\alpha)$ is 0 if the total number of particles at time t is less than α and 1 otherwise, its expectation value gives precisely $P(\alpha,t)$.

As the Θ function is dimensionless, it follows straightforwardly from Eq. (4) that the scaling dimensions [18] of

$P(\alpha,t)$ and $\psi(\mathbf{x},t)$ are equal. Since Eqs. (2) and (3) imply that $P(\alpha,t)\sim t^{-\delta}$, we conclude that $-\delta$ gives the scaling dimension $[\psi(\mathbf{x},t)]$ of the field ψ, expressed as a power of an arbitrary timescale T, i.e., that

$$[\psi]=T^{-\delta}. \tag{5}$$

Thus δ can be simply computed as the negative of the total dimension of the response field in the field theory. This constitutes the main result of this paper and provides a practical method for determining δ. Note that δ is well defined in the field theory and is independent of α, even though the survival probability $P_s(t)$ is always unity. We now use Eq. (5) to determine scaling relations, and/or the value of δ, for different theories with absorbing states.

IV. APPLICATIONS

A. Reggeon field theory

The dynamical Lagrangian associated with Eq. (1) is [4,13]

$$\mathcal{L}=\int d^dx\ dt\left[\frac{D}{2}\psi^2\phi+\psi(\partial_t\phi-\nabla^2\phi-r\phi+u\phi^2)\right]. \tag{6}$$

The fields and parameters can be rescaled so as to make the coefficients of the two nonlinear terms the same. This makes the Lagrangian invariant under the (permutation) transformation $\phi(\vec{x},t)\leftrightarrow\psi(\vec{x},-t)$. It is easy to check that in order for the Lagrangian to be dimensionless, we need $[\phi\psi]_0\sim T^{-dz/2}$, where $[\ \ ]_0$ denotes the mean-field or engineering dimension. To account for the anomalous dimension [13] coming from the diagrammatic corrections, we write $L^2\sim T^z$, where L is a length scale, thereby defining the dynamical exponent [19] z, and $[\phi\psi]\sim T^{-dz/2+\mu+\gamma}$, where μ and γ are defined as the anomalous dimensions [18] of the fields ϕ and ψ respectively.

The permutation symmetry yields $[\phi]=[\psi]$, and consequently $\mu=\gamma$, whereupon we conclude that $[\psi]=-\delta=-dz/4+\gamma$. We are now in a position to relate δ to the exponent η, an exponent commonly determined in numerical studies of discrete models with absorbing states. This exponent is defined by the expression $N(t)\sim t^{\eta}$ for the total number of particles $N(t)$ in the system at asymptotically large times t resulting from a single-particle initial seed [12,20]. Now in the field-theory representation, $N(t)$ is clearly given by $N(t)\sim\langle\int d^dx\ \phi(x,t)\psi(0,0)\rangle\sim t^{\mu+\gamma}$, whereupon

$$4\delta+2\eta=dz. \tag{7}$$

This is a well-known scaling law for RFT [12,3], previously derived from a self-duality relation [12,3]. Our derivation follows directly from the symmetry of the Lagrangian and does not require the self-duality property.

B. Systems without the RFT symmetry

Suppose now that there is an extra term in the Lagrangian that breaks the $\phi\leftrightarrow\psi$ symmetry. Systems with infinite numbers of absorbing states [21], which occur in some of the same physical contexts (notably catalysis and epidemiology) that give rise to models in the RFT universality class, constitute a familiar example. In this particular case, the right-hand side of the Langevin equation (1) acquires [21] an extra term proportional to $\phi(\mathbf{x},t)\exp[-w_1\int_0^t ds\ \phi(\mathbf{x},s)]$, i.e., a term nonlocal in time, for some constant w_1. This produces in the Lagrangian (6) an extra term proportional to the product of this new contribution and the response field ψ; this extra term clearly destroys the $\phi\leftrightarrow\psi$ symmetry.

It has been argued that in this case the scaling relations for initial seed problems require modification [22]. In the corresponding microscopic models, a new universal exponent δ' is defined through the asymptotic time dependence of the density $\rho(t)$ of particles in the occupied region resulting from an initial seed, averaged only over the surviving trials. It follows from the definitions of the exponents η, δ, and z that $\rho(t)\sim\langle N(t)\rangle_{surv}/L^d\sim t^{\delta+\eta-dz/2}$ and hence that

$$2(\delta+\delta')+2\eta=dz. \tag{8}$$

This is precisely the scaling relation proposed for systems with an infinite number of absorbing states [22]. It follows further from the definition of $\rho(t)$ that in general $\rho(t)\sim\langle\phi\psi\rangle/P(\alpha,t)$. When the permutation symmetry is restored, this gives $\delta'=\delta$ and we recover Eq. (7).

C. Voter model and compact directed percolation

The voter model and compact directed percolation [5,23] are models that have absorbing states and are known to belong in a universality class distinct from RFT. The physical reason for this is that the dynamics in these models takes place only at the boundaries separating empty regions and occupied (''infected'') regions, and not in the interior of occupied clusters, as it does in models of the RFT class. This difference changes the critical exponents and hence the universality class. A Langevin equation describing this situation was proposed in [23,24]:

$$\partial_t\phi(x,t)=\lambda\nabla^2\phi(x,t)+[\phi(1-\phi)]^{1/2}\eta(x,t), \tag{9}$$

where the noise η is defined as in Eq. (1). The associated Lagrangian is

$$\mathcal{L}=\int d^dx\ dt\left[\frac{D}{2}\phi(1-\phi)\psi^2-\psi(\partial_t\phi-\lambda\nabla^2\phi)\right]. \tag{10}$$

Note that this model has two uniform absorbing states: $\phi(x)=0$ and $\phi(x)=1$. The Lagrangian is also invariant under the transformation $\phi\leftrightarrow1-\phi$, $\psi\rightarrow-\psi$.

Counting powers as in RFT yields $[\phi\psi]=T^{-zd/2+\mu+\gamma}$, where, as before, μ and γ are the anomalous dimensions of the ϕ and ψ fields, respectively. It follows from the $\phi\leftrightarrow1-\phi$, $\psi\rightarrow-\psi$ symmetry, moreover, that the field ϕ must be dimensionless, whereupon $[\psi]=T^{-zd/2+\gamma}$. The exponents γ and z can be obtained perturbatively from diagrammatic corrections to the propagator. In fact, however, it is a trivial matter to check that there are no such corrections to the propagator coming from the nonlinear terms in Eq. (10). Therefore, γ and z take their mean field values 0 and 1, respectively. Putting all this together yields $\delta=d/2$. Thus one obtains the known exact results for the voter model [5,23]: $\delta=1/2$ in $d=1$ and $\delta=1$ in $d=2$.

D. Dynamical percolation

Percolation clusters can be generated from the discrete dynamical model known as *dynamical percolation*, proposed by Grassberger [25] to describe the spreading of epidemics or forest fires [6,7]. Expressed in the language of forest fires, the idea is that a propagating fire front leaves behind a cluster of burned trees that cannot burn again. Below a critical value of the control parameter of the system, the fire burns itself out in finite time, putting the system in the absorbing phase and leaving behind a finite cluster of burned trees. Right at the critical value, the number of trees in the burned cluster diverges in the thermodynamic limit, while above criticality the system is in the active phase, characterized by a fire that survives indefinitely and an ever-expanding burned cluster. The cluster generated at criticality is precisely the (fractal) percolation cluster for the lattice in question.

The discrete model can be written as a field theory characterized by the Lagrangian [7]

$$\mathcal{L}=\int d^dx\ dt\left[\frac{D}{2}\psi^2\phi-\psi\left(\partial_t\phi-\nabla^2\phi-r\phi+w\phi\int_0^t ds\ \phi(s)\right)\right]. \tag{11}$$

As before, naive power counting analysis straightforwardly yields the result $[\phi\psi]_0\sim T^{-dz/2}$. It is easy to verify, moreover, that the Lagrangian (11) satisfies the symmetry [7(c)] $\psi(\vec{x},t)\leftrightarrow-\int_0^t ds\ \phi(\vec{x},s)$ or, equivalently, $\phi(\vec{x},t)\leftrightarrow-\partial\psi(\vec{x},t)/\partial t$. From this symmetry it follows immediately that $[\phi]_0\sim T^{-dz/4-1/2}$, $[\psi]_0\sim T^{-dz/4+1/2}$, and the upper critical dimension d_c is 6, the known result for dynamical percolation [25]. When corrections to the naive scaling coming from diagrams are introduced, one gets $[\phi]\sim T^{-dz/4-1/2+\mu}$ and $[\psi]\sim T^{-dz/4+1/2+\gamma}$, where, as before, μ and γ are the anomalous dimensions of the fields. Using these expressions, the symmetry, and our conjecture we obtain after some simple algebra

$$\eta+2\delta+1=\frac{dz}{2} \tag{12}$$

as a scaling law for dynamical percolation. Checking that this scaling relation coincides with the one derived by Grassberger in Ref. [25], namely, $\gamma=2(\nu d/2-\beta)$, is a simple task; one need only note the following correspondence between the exponents considered here and the ones defined in [25]: $\delta=\beta/\tau$, $z=2\nu/\tau$, and $\eta=\gamma/\tau-1$.

V. CONCLUSIONS

In discrete models with absorbing states, an exponent δ can be readily defined as governing the asymptotic decay of the survival probability at criticality. In field theories with continuous variables, however, the survival probability is always unity and it is unclear that δ can even be defined sensibly. Here we have proposed a rationalization of this problem, by defining δ in terms of the way the probability distribution piles up around the absorbing state. We have presented arguments to support the validity of this definition, which we then used to derive δ directly from field theory. Applying this method to a number of different cases of interest, we reproduced correct scaling relations in all cases and were able to compute known values of δ exactly in others. This strongly supports the legitimacy of our construction. Together with our definition, standard field-theoretic techniques can be used to derive scaling relations and critical exponents for systems with absorbing states more easily and systematically than in other methods. Applications of these ideas to field theories for problems with multiplicative noise [8] and to Peliti's field theory [24] for the reaction $A+A\rightarrow A$ will be addressed elsewhere.

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